Applied mathematics and mathematical methods

Problems handled in the exam on Thursday 22nd of June, 2017, 8:00, BM-116.
The retake exam is proposed to take place on Friday 29th of September, 2017, 8:00.

1. The set theory, a relation, an equivalent relation, a function and its types (injection, surjection, bijection).
2. 5th postulate of Euclid. Euclidean, elliptic and hyperbolic geometries.
3. The Bertrand paradox.
4. Anchored/bounded and free vectors.
5. The dot/inner/scalar product of 2-, 3- and *n*-dimensional vectors.
6. An inverse matrix and a Cramerian sale (=system of algebraic linear equations).
7. The Kronecker-Capelli theorem.
8. An algebraic/matrix eigenproblem: a characteristic polynomial, an eigenvalue, an eigenvector, a spectrum.
9. Linear combination and independence of vectors.
10. A diagonalization of a matrix having the full spectrum.
11. A polynomial collocation (and interpolation) in Stevin/natural and Lagrange bases, theorem on the uniqueness.
12. The Runge phenomenon and Chebychev nodes.
13. A least-square approximation/fit and its error (aka a deviation).
14. A trigonometric collocation and interpolation.
15. A power series and its convergence.
16. The Euler expansion (1777) for $r=b/(1+\varepsilon\cdot\cos\theta)$.
17. A Fourier series and Euler-Fourier coefficients.
18. The Dirichlet criterion, Bessel minimalization property and Parseval identity.
19. A power/solution to an ODE1.
20. A separable ODE1. Finding a particular solution to a nonhomonogenous ODE1 by the variation of a constant and by the examination of expected shape.
21. ODE1 describing an exponential growth, a RC circuit.
22. The Newton law of cooling (1701).
23. The logistic/Verhulst equation (1838).
24. ODE2 describing MSD (mass,spring,damper) system and RLC (resistor,inductor, capacitor) circuit.
25. A matrix exponential and a system of ODE1s ($v' = Mv$).
26. An absolute/fundamental solution to $u''(x) = \delta(x)$, where δ is Dirac delta.
27. A PDE, the Cauchy-Kovalevskaya theorem.
28. A general PDE2.*n* (=partial differential equation of the order 2 in *n* variables), a recognition of the type of CC-LPDE2.*n* (linear PDE with constant coefficients).
29. Elliptic PDE2.*n* : a Laplace equation ($\nabla^2 u = 0$), a Poisson equation ($\nabla^2 u = b$).
30. Parabolic PDE2.1: an one-dimensional heat (diffusion) equation ($u_t = c\cdot u_{xx}$).
31. Hyperbolic PDE2.1: a one-dimensional string/wave equation ($u_{tt} = c^2\cdot u_{xx}$).
32. d'Alembert solution (1747) to 1D-wave eqn: $u(t,x) = \{s(x-ct)+s(x+ct)\}/2+ \int_{x-ct}^{x+ct} d(\xi)d\xi/(2c)$, if $u(0,x)=s(x)$ and $u_t(0,x)=d(x)$.
33. Fourier solution (or solution produced by the factorizing, or separation of variables) to 1D-wave eqn: $u(t,x) = \Sigma_{k=0..\infty}\, T_k(t)\cdot X_k(x)$, where T_k and X_k are solutions to equations $X_k''=X_k$, $T_k''=c^2\cdot T_k$ (and bundary conditions are taken into account).
34. Derivation of the string equation based on Newton law and Hooke law.
35. Derivation of the wave equation based on a random walk.

Adam Marlewski, 2017-05-30, 14:24